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Habilitation Thesis Reviewer's Report

Masaryk University	
Faculty	Faculty of Science
Procedure field	Mathematics - Geometry
Applicant	Mgr. Lenka Zalabová, Ph.D.
Applicant's home unit, institution	University of South Bohemia in České Budějovice
Habilitation thesis	Filtered Manifolds with Distinguished Transformations and Transformation Groups
Reviewer	Peter J. Olver, Ph.D.
Reviewer's home unit, institution	School of Mathematics University of Minnesota Minneapolis, MN 55455 USA

Lenka Zalabová's thesis comprises a detailed study of the differential geometry of manifolds with distinguished structure groups, with particular emphasis on the case of parabolic geometries. Applications to nonholonomic mechanics are also considered, and the author explains how the abstract machinery has some impact on the latter area.

The first part of the thesis is of a theoretical nature, and concerned with the symmetries (automorphisms) and structure theory of parabolic geometries, which are an important subclass of general Cartan geometries. Particular attention is paid to the structures over filtered manifolds. For simplicity, the thesis restricts attention to what are called regular parabolic geometries, a technical condition involving the homogeneity properties of the curvature tensor. There is also a restriction to automorphisms which have a natural tangent action, which serves to specify the class of symmetries and generalized symmetries to be analyzed. These definitions are justified as forming natural generalizations of the well-developed topic of symmetric spaces, which serves to motivate the more challenging geometries considered here.

The key results concern the existence and classification of such (generalized) symmetries, which form the topics of Theorems 1 and 2 (I am using the numbering in the introductory chapter). Interestingly, in contrast to affine geometries, as the author has found, these geometries need not be smooth. Indeed, Theorem 3 gives an explicit characterization of such smooth systems based on the existence of symmetries. Theorem 4 proves the interesting result establishing, under the assumption of "prolongation rigidity", necessary conditions for the existence of an invariant Weyl connection, based on the non-existence of a unit eigenvalue of the adjoint action, thereby generalizing earlier known special cases. Theorem 5 establishes, under similar conditions, the equivalence of symmetric geometries and the existence of what the author calls f-invariant Weyl connections. Next, the particular case of

CR geometries leads to several new results in this well-studied subject. Theorem 6 gives a complete classification of symmetries and compatible metrics for the flat model. In the case of non-flat almost CR structures, Theorem 7 shows how the existence of symmetries places important restrictions on the Weyl curvature and Nijenhuis torsion tensors. The author further introduces the notion of a reflexion space, modelled on the concept of a symmetric space, and characterizes them in Theorems 8 and 9, while Theorem 10 deals with the existence of compatible pseudo-Riemannian metrics. Finally, Theorem 11 finds strict upper bounds on the dimension of the symmetry algebra in the almost-quaternionic case.

The second part of the thesis shifts gears to analyze certain geometric control problems, including snake robots and rolling disks. The thesis concludes with several numerical computations of optimal trajectories for the vertical rolling disk problem. I found the connections between the two parts of the thesis somewhat tenuous, and incompletely explained. More details on how the geometric control problems lead to particular parabolic geometries would be very useful. At the same time, using these relatively simple examples to illustrate the abstract and at times challenging constructions in the first part of the thesis would have greatly aided in the readers comprehension of these results. In particular, do any of the previous Theorems 1-10 have anything to say about the control problems and their analysis?

I should remark that I found some of the material in the introductory chapter a bit hard to follow, with a number of concepts and notations not defined before being discussed. The ensuing papers serve to flesh out the survey of results covered in the introductory chapter.

The author has clearly devoted a lot of effort to these complicated and nontrivial issues, and has thereby shed new light on the structures underlying parabolic geometries. For this, she is to be commended

Let me now discuss the individual papers in the body of the thesis.

The first paper, solo authored, discusses how to adapt the concept of a locally symmetric space to parabolic contact geometries, showing that this requires them to be torsion-free and, when the symmetry is non-involutive, locally flat. In the non-flat case, under certain conditions, there can exist only one symmetry. The second paper, co-authored with Gregorovic, studies local automorphisms (symmetries) of parabolic geometries, characterizing and classifying them using the curvature tensor and Weyl connections. The paper contains a number of extensive tables detailing the various possibilities, that will be an excellent resource for future researchers wishing to further develop these studies. The third paper, also co-authored with Gregorovic, studies symmetric CR geometries of so-called hypersurface type, proving that they are either flat or homogeneous, and, in the latter case, admit a compatible pseudo-Riemannian metric. Paper 4, co-authored with Krugkilov and Winther, proves the above-mentioned results concerning maximally and submaximally symmetric almost quaternionic structures. Finally paper 5, co-authored with Hrdina, turns to the control-theoretic aspects of the thesis, concentrating on the nilpotent approximation to a certain generalized trident mechanism. Details concerning particular solutions in special cases are developed.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

Can you give a more convincing justification of how parabolic geometries show up in the optimal control problems you are considering? In particular, does there exist a list of parabolic geometries that have or might appear in control theoretic problems. Given a control problem, is there a systematic procedure for determining the relevant underlying geometry?

Can you illustrate some of the geometric constructions in the first part of the thesis by one of the parabolic geometries appearing in the optimal control problems considered in the second part?

Do the general Theorems you have proved concerning parabolic geometries have any applications to control theory? If so, explain in detail.

What are the physical interpretations of the symmetries, Weyl tensor, and pseudo-Riemannian structure in the control-theoretic setting?

Conclusion

The habilitation thesis entitled "Filtered Manifolds with Distinguished Transformations and Transformation Groups" by Mgr. Lenka Zalabová, Ph.D. ***fulfils*** the requirements expected of a habilitation thesis in the field of Mathematics - Geometry.

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